

# RESONANCES AND UNITARITY IN COMPOSITE HIGGS MODELS

D. ESPRIU

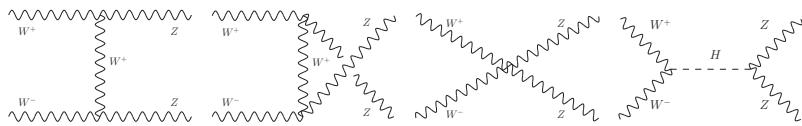
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The scattering of longitudinally polarized  $W$  bosons in extensions of the Standard Model with anomalous Higgs couplings to the gauge sector and higher order  $O(p^4)$  operators is considered. The modified couplings should be thought as the low energy remnants of some new dynamics involving the electroweak symmetry breaking sector. By imposing unitarity and causality constraints on  $W_L W_L$  scattering amplitudes we relate the possible values of the effective couplings to the presence of new resonances above 300 GeV. We investigate the properties of these new resonances and their experimental detectability.

## 1 Introduction

We know that in the SM the Higgs boson unitarizes  $W_L W_L$  scattering. Consider e.g. the process  $W_L^+ W_L^- \rightarrow Z_L Z_L$ . The first 3 diagrams are fixed by gauge invariance, but we can contemplate



different Higgs-gauge boson couplings in the last one. If any of these couplings are different from the Standard Model (SM) values, the careful balance necessary for perturbative unitarity is lost. For  $s \gg M_W^2$  the amplitude in the SM goes as

$$\frac{s}{v^2} \frac{M_H^2}{s - M_H^2} \sim \frac{M_H^2}{v^2}, \quad (1)$$

but on dimensional grounds it should go as

$$\frac{s}{v^2} \frac{s}{s - M_H^2} \sim \frac{s}{v^2}. \quad (2)$$

This is indeed what happens after any modification of the Higgs couplings and produces non-unitary amplitudes. In short the SM value is precisely tuned to preserve unitarity.

Adding new effective operators typically spoils unitarity too

$$\mathcal{L}_{SM} \rightarrow \mathcal{L}_{SM} + \sum_i a_i \mathcal{O}_i. \quad (3)$$

New physics may produce either type of modifications. What can the requirement of unitarity in  $W_L W_L$  scattering tell us about possible anomalous couplings in the electroweak (EW) sector?

## 2 Parametrizing composite Higgs physics

A light Higgs boson with mass  $M_H \sim 125$  GeV is coupled to the EW bosons according to<sup>1</sup>

$$\begin{aligned} \mathcal{L}_{\text{eff}} \supset & -\frac{1}{2} \text{Tr} W_{\mu\nu} W^{\mu\nu} - \frac{1}{4} \text{Tr} B_{\mu\nu} B^{\mu\nu} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} + \sum_i a_i \mathcal{O}_i \\ & + \left[ 1 + 2a \frac{h}{v} + b \frac{h^2}{v} \right] \frac{v^2}{4} \text{Tr} D_\mu U^\dagger D^\mu U - V(h) \end{aligned} \quad (4)$$

$$U = \exp(i \omega \cdot \tau/v) \quad D_\mu U = \partial_\mu U + \frac{1}{2} i g W_\mu^i \tau^i U - \frac{1}{2} i g' B_\mu^i U \tau^3 \quad (5)$$

A non-linear realization is used. Setting  $a = b = 1$  and  $a_i=0$  exactly reproduces the SM interactions.

The  $\mathcal{O}_i$  are a full set of  $C$ ,  $P$ , and  $SU(2)_L \times U(1)_Y$  gauge invariant,  $d = 4$  operators<sup>2</sup> (of  $O(p^4)$  in the chiral language) that along with the couplings  $a, b$  parameterize the low-energy effects of an extended high-energy EW symmetry breaking sector (EWSBS). If we assume that the EWSBS is custodially preserving the relevant operators for  $W_L W_L$  scattering are

$$\mathcal{L}_4 = a_4 (\text{Tr} [V_\mu V_\nu])^2, \quad \mathcal{L}_5 = a_5 (\text{Tr} [V_\mu V^\mu])^2, \quad V_\mu = (D_\mu U) U^\dagger. \quad (6)$$

The  $a_i$  could be functions of  $\frac{h}{v}$ . The contribution of these  $d = 4$  operators to  $W_L^{(\mu)} W_L^{(\nu)} \rightarrow Z_L^{(\rho)} Z_L^{(\sigma)}$  scattering is given via the Feynman rule

$$ig^4 [a_4 (g^{\mu\sigma} g^{\nu\rho} + g^{\mu\rho} g^{\nu\sigma}) + 2a_5 g^{\mu\nu} g^{\rho\sigma}] \quad (7)$$

Experimentally there are by now solid indications that the Higgs particle couples to the  $W, Z$  very similarly to the SM rules. Let us assume for the time being that  $a = b = 1$  *exactly*. Then

$$\mathcal{L}_{\text{eff}} \simeq \mathcal{L}_{\text{SM}} + a_4 (\text{Tr} [V_\mu V_\nu])^2 + a_5 (\text{Tr} [V_\mu V^\mu])^2 \quad (8)$$

$a_4$  and  $a_5$  represent anomalous 4-point couplings of the  $W$  bosons due to an extended EWSBS that however does not manifest with  $O(p^2)$  couplings being noticeably different to the ones in the SM. These anomalous couplings will lead to violations of perturbative unitarity as they lead to amplitudes that grow<sup>1,2</sup> as  $s^2$ .

## 3 Unitarity and resonances

Violations of unitarity are cured by the appearance of new particles or resonances. We can now use well-understood unitarization techniques to constrain these resonances and the effective couplings  $\{a_i\}$ . First, let us recapitulate

- The Higgs particle unitarizes amplitudes in the SM, where  $a = b = 1, \{a_i\} = 0$ .
- The theory is renormalizable without the  $\{a_i\}$  if  $a = b = 1$ .
- If present, the  $\{a_i\}$  will then be finite non-running parameters.

We would like to

- Determine how much room is left for the  $a_i$ .
- Find possible additional resonances required to restore unitarity.
- Should we have already seen any of these resonances?
- To what extent an extended EWSBS is excluded by current data?

We advance some answers:

- Yes, there may be new resonances with relatively light masses and narrow widths.
- No, we should not have seen them yet. Their signal is too weak.
- Looking for the resonances is an efficient (albeit indirect) way of setting constraints on anomalous triple and quartic gauge couplings (i.e. the  $a_i$ ).

Let  $t_{IJ}(s)$  be a partial wave derived from the  $W_L W_L \rightarrow Z_L Z_L$  amplitude. Unitarity requires

$$\begin{aligned} \text{Im } t_{IJ}(s) &= \sigma(s)|t_{IJ}(s)|^2 &+ \sigma_H(s)|t_{H,IJ}(s)|^2 \\ &\text{Elastic} & \text{Inelastic} \\ &WW \rightarrow WW & WW \rightarrow hh \end{aligned} \quad (9)$$

where  $\sigma$  and  $\sigma_H$  are phase space factors. Given a perturbative expansion

$$\begin{aligned} t_{IJ} &\approx t_{IJ}^{(2)} + t_{IJ}^{(4)} + \dots \\ &\text{tree} & \text{one-loop} \\ &+ a_i \text{ terms} \end{aligned} \quad (10)$$

we can require unitarity to hold exactly by using the inverse amplitude method (IAM) to define

$$t_{IJ} \approx \frac{t_{IJ}^{(2)}}{1 - t_{IJ}^{(4)}/t_{IJ}^{(2)}} \quad (11)$$

for non-coupled channels.<sup>3</sup> Several analyticity assumptions are implied in the above derivation.

Unitarization of the amplitudes may result in the appearance of new heavy resonances associated with the high-energy theory ( $t_{00} \rightarrow$  Scalar isoscalar  $t_{11} \rightarrow$  Vector isovector  $t_{20} \rightarrow$  Scalar isotensor). We will search for poles in  $t_{IJ}(s)$  up to  $4\pi v \sim 3$  TeV (domain of applicability of the effective theory). Physical resonances will be required to have the phase shift pass through  $+\pi/2$ . This method is known to work remarkably well in strong interactions.

Is this unitarization method unique? Certainly not. Many methods exist: IAM, K-matrix approach, N/D expansions, Roy equations,.... While the quantitative results differ slightly, the gross picture does not change. For a detailed discussion of the different procedures see.<sup>3</sup>

#### 4 Calculation and results for $a = b = 1$

Most studies concerning unitarity at high energies are carried out using the Equivalence Theorem (ET). This is understandable as calculations simplify enormously<sup>4</sup>

$$A(W_L^+ W_L^- \rightarrow Z_L Z_L) \rightarrow A(\omega^+ \omega^- \rightarrow \omega^0 \omega^0) + O(M_W/\sqrt{s}) \quad (12)$$

For a light Higgs one needs to include tree-level Higgs exchange as well. Then one could make use of the well known chiral lagrangian techniques to derive the amplitudes and compare with experiment, including the Higgs as an explicit resonance. However for  $s$  not too large (which obviously is now an interesting region) the simplest version of the ET may be too crude an approximation and we shall use as much as possible exact amplitudes.

However, a full calculation of the one-loop contribution for the  $W_L W_L \rightarrow Z_L Z_L$  process,  $t_{IJ}^{(4)}$ , in particular for arbitrary values of  $a$  and  $b$  is beyond question. Only one complete calculation

exists due to Denner and Hahn<sup>5</sup> for the SM case and it is available only numerically; not suitable for unitarity analysis. We can take a shortcut. The optical theorem implies the *perturbative* relation

$$\begin{aligned} \text{Im } t_{IJ}^{(4)}(s) &= \sigma(s)|t_{IJ}^{(2)}(s)|^2 + \sigma_H(s)|t_{H,IJ}^{(2)}(s)|^2 \\ \text{one-loop} & \qquad \qquad \qquad \text{tree} \end{aligned} \quad (13)$$

For the *real part*, note that

$$\begin{aligned} \text{Re } t_{IJ}^{(4)} &= a_i\text{-dependent terms} + \text{real part of loop calculation} \\ &\approx a_i\text{-dependent terms} \quad (\text{for large } s, a_i) \end{aligned} \quad (14)$$

We approximate the *real part of loop contribution* with one-loop Goldstone boson amplitudes using the ET. The other contributions are computed exactly. See<sup>1</sup> for details.

Are there resonances? To answer this question we must search for poles in the second Riemann sheet — the phase shift must go through  $+\pi/2$  at the resonance. Are there any physically acceptable resonances? This question is answered in the positive. If one looks for resonances with masses below 3 TeV they are present for virtually any value of  $a_4$  and  $a_5$ , except for values very close to zero (i.e. very close to the SM).

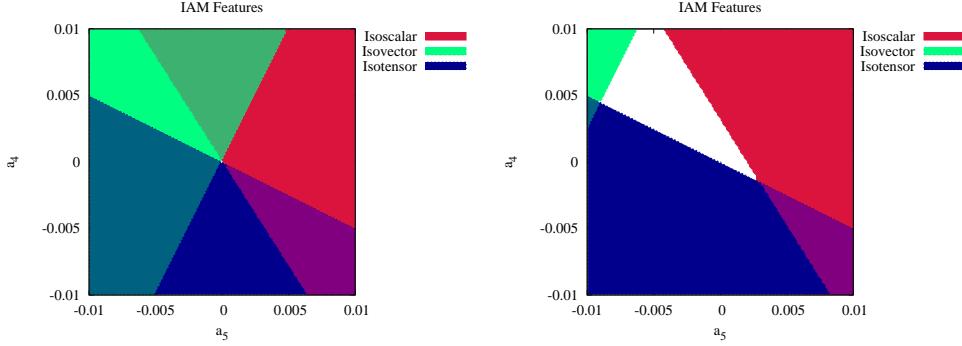
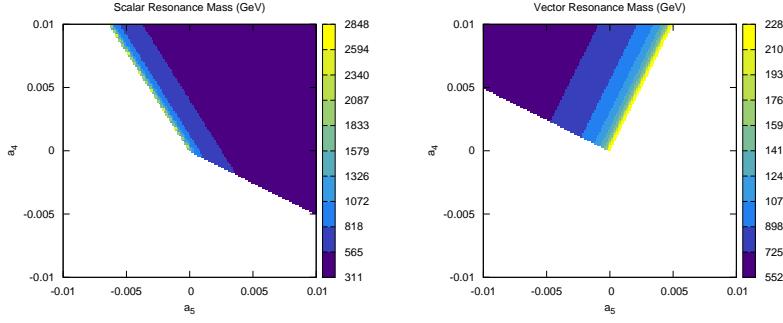


Figure 1 – Left: for  $a = b = 1$  regions in  $a_4 - a_5$  leading to acceptable resonances. The red (green) region corresponds to acceptable isoscalar (isovector) resonances. The blue-shaded area leads to acausal resonances and the corresponding values for  $a_4$  and  $a_5$  are unphysical — they cannot be realized in any effective theory with a meaningful UV completion. Only a extremely small set of  $a_4 - a_5$  parameters (very close to the SM values —zero-nearly invisible in the figure) do not lead to new resonances below 3 TeV. Right: same but now we impose that the resonances should be found below 600 GeV. If not present, the range of values for the anomalous couplings still acceptable (white area) is much enlarged. This could possibly represent the present experimental situation according to the present analysis.

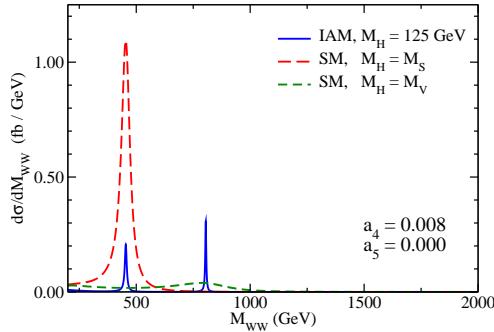
#### 4.1 Properties of the new resonances

In the next figure we show the masses that are obtained in the scalar and vector channels. As we see, by varying the values of  $a_4 - a_5$  we obtain masses in the regions  $M_S \sim 300 - 3000$  GeV,  $M_V \sim 550 - 2300$  GeV. This means that relatively light masses are possible in extended EWSBS leading to appropriate values of the  $d = 4$  effective couplings. Observing or excluding these resonances is thus an indirect way of measuring these effective couplings. Note that this analysis is independent of the precise nature of this sector because only general arguments (locality, unitarity,...) have been used. We have similar plots for the widths but we will not present them here due to space reasons. The resonances are generally speaking narrow:  $\Gamma_S \sim 5 - 120$  GeV,  $\Gamma_V \sim 2 - 24$  GeV.



#### 4.2 Visibility of the resonances

The next question is whether these resonances are detectable. The answer is that this is impossible with the present experimental statistics. To see this point clearly we show the signal of two of the resonances predicted by unitarity: one scalar and one vector. They correspond to the values for  $a_4$  and  $a_5$  indicated in the figure. For these values both one scalar and one vector resonances are present (the vector one is heavier). We compare the strength of the signal of the scalar resonance to the one corresponding to a SM Higgs with the same mass. Resonances could still be there, but would give a small signal. This signal is undetectable at present and will necessitate at least 10 times more statistics. In addition this signal would only be present in the  $WW \rightarrow WW$  or  $WW \rightarrow ZZ$  channels. The large contribution that the SM Higgs represents leaves little room for additional resonances.



## 5 Moving away from the SM Higgs couplings

What if the  $hWW$  couplings are not exactly the SM ones? Nothing prevents us from carrying out the same programme for arbitrary values of the Higgs-to- $WW$  couplings  $a$  and  $b$ . The resulting effective theory is non-renormalizable and the  $a_i$  will be required to absorb the additional divergences<sup>6</sup>

$$\delta a_4 = \Delta_\epsilon \frac{1}{(4\pi)^2} \frac{-1}{12} (1 - a^2)^2 \quad (15)$$

$$\delta a_5 = \Delta_\epsilon \frac{1}{(4\pi)^2} \frac{-1}{24} \left[ (1 - a^2)^2 + \frac{3}{2} ((1 - a^2) - (1 - b))^2 \right] \quad (16)$$

We can repeat the same unitarization procedure as for  $a = b = 1$  and search for resonances. The results are shown in the following figure. The characteristics of the resonances tend smoothly to the  $a = 1$  case ( $hWW$  coupling as in the SM). Resonances tend to be slightly heavier and broader than for  $a = 1$ . The parameter  $b$  is only marginally visible in the widths (not shown). There are constraints on vector masses from  $S, T, U$  parameter constraints in some models.<sup>7</sup>

As in the  $a = 1$  case the signal is always much lower<sup>1</sup> than the one for a Higgs of the same mass. For  $a = 1$  typically  $\sigma_{\text{resonance}}/\sigma_{\text{Higgs}} < 0.1$ , now  $\sigma_{\text{resonance}}/\sigma_{\text{Higgs}} \simeq 0.2$ .

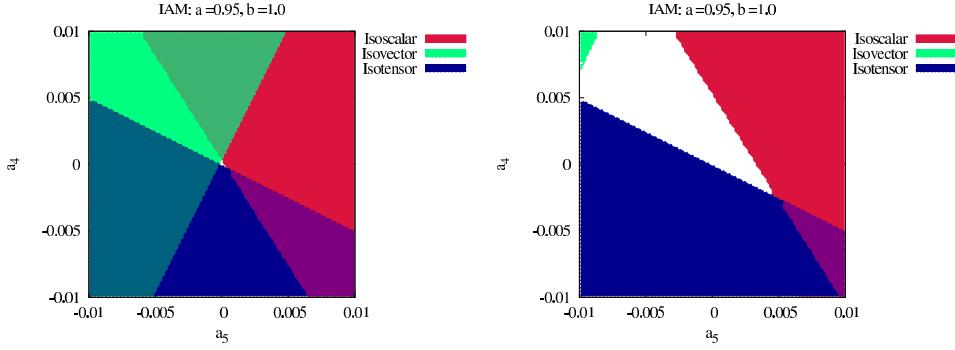


Figure 2 – Exclusion zones and bounds on  $a_i$  for  $a = 0.95$  ( $b = a^2$ ). Left: in color the excluded region in  $a_4 - a_5$  parameter space if no resonance is found  $< 3$  TeV. Right: in color the excluded regions if no resonance exists below  $M_R = 600$  GeV. Note that the allowed white areas are slightly larger than in the case  $a = b = 1$

To summarize, the situation for  $a < 1$  is not radically different from  $a = 1$ . Resonances (particularly in the vector channel) are slightly more difficult to appear. They tend to be slightly heavier and broader and they give a slightly larger experimental signal.

This situation changes drastically for  $a > 1$ . ‘Something’ happens when  $a > 1$ . Most of the resonances disappear and in fact most of parameter space is excluded on causality and unitarity grounds. We have no space left to explain the reasons of this radical change of behaviour here and recommend the interested reader to examine our references.<sup>8,1</sup> From a technical point of view, this drastic modification is associated to the change of sign of  $t^{(2)}$  when  $a > 1$ .

Let us summarize our main points. Unitarity is a powerful constraint on scattering amplitudes. The validity is well tested in other physical situations. Even in the presence of a light Higgs, unitarization can help constrain anomalous couplings by helping predict heavier resonances. An extended EWSBS would typically have such resonances even in the presence of a 125 GeV Higgs. However the properties of the resonances are radically different from the ‘standard lore’. Limited by statistics, existing LHC searches do not yet probe the IAM resonances.

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